SECONDARY/ESL PROGRAM UNIT TEMPLATE

Unit Topic: Quadratic Functions

Unit Designer: Nora Evans

Subject Area(s) / Grade Level(s): Advanced Algebra 2 / Grades 10-12

Overview of Unit (Including information about the class/students, what the unit is about, when in the instructional sequence this unit will take place, why the unit is important within the context of the curriculum and for adolescent students, how you will help students see the importance and relevance of this content to their lives)

Class Description:
In *Advanced Algebra 2*, the students learn at a slower pace the algebra content of the *Algebra 2/Trigonometry* class (the old *Honors Algebra 2*). The class is required for graduation with a high school degree, and it is one of the three mathematics high school classes a student can choose for the SOL examination in Mathematics in high school.

Students: Several students took the class before but did not get credit because of poor performance on the Power Standards (essential knowledge a student must demonstrate to advance to *Algebra 3*).
No student is expected to take the Algebra 2 SOL. The students either passed *Algebra 1* SOL or will take it instead of *Algebra 2*.
All the students in the class are proficient in English. One student has special accommodations (needs the tests read to him, but not the class material).

Content. In the *Quadratic Functions* unit, the students investigate the quadratic function family. Quadratic functions are often used in Newtonian Mechanics to describe the motion of objects with constant acceleration such as footballs, stopping distance. They are also used in economics for profit calculations.
In the context of the high school mathematics, the study of quadratic functions builds mathematical maturity by practicing the skills learned in *Algebra 1* and in the study and analysis of linear functions, in the context of a more complex function. Students learn to factor second degree polynomials, a skill needed later in rational functions. In the *Quadratic Functions* unit, apply previously studied transformations to a new function, deepening their understanding of the subject, as well as building a set of tools to tackle complex functions they will study in Math Analysis.

Position in the Instructional Sequence. The *Quadratic Functions* unit follows the study of linear functions, and it is followed by the polynomial functions.

Established Goals (VSOLs and/or National Standards)

VSOL. Algebra 2

AII.1 The student will
~~a) add, subtract, multiply, divide, and simplify rational algebraic expressions;~~
~~b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; and~~
c) factor polynomials completely in one or two variables.

AII.3 The student will solve
~~a) absolute value linear equations and inequalities;~~
b) quadratic equations over the set of complex numbers;
~~c) equations containing rational algebraic expressions; and~~
~~d) equations containing radical expressions.~~

AII.6 For absolute value, ~~square root, cube root, rational,~~ polynomial~~, exponential, and logarithmic functions,~~ the student will
a) recognize the general shape of function families; and
b) use knowledge of transformations to convert between equations and the corresponding graphs of functions.

AII.7 The student will investigate and analyze ~~linear,~~ quadratic~~, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic~~ function families algebraically and graphically. Key concepts include:
a) domain, range~~, and continuity;~~
b) intervals in which a function is increasing or decreasing;
c) extrema;
d) zeros;
e) intercepts;
~~f) values of a function for elements in its domain;~~
~~g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;~~
~~h) end behavior;~~
~~i) vertical and horizontal asymptotes;~~
~~j) inverse of a function; and~~
~~k) composition of functions algebraically and graphically.~~

AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

Stage One: Essential Questions & Unit Learning Targets (UKDs)

Meaning	
Overarching Concepts: Representations, Divide and Conquer	
Students will explore these ESSENTIAL QUESTIONS…	**Students will Understand THAT…**
(EQ1) Is a picture worth a thousand words? How do successful mathematicians use graphical representations to understand and solve problems? (EQ2) What do good problem solvers do to decompose hard problems into several simpler problems? (EQ3) How do skilled problem solvers decide which representation to use?	(U1) Students will understand that the graph of a function is a succinct and powerful representation of the function's algebraic properties and that it is a useful tool to solve a variety of problems. (U2) Students will understand that hard problems can be solved easily when reduced to a succession of simpler problems by using working backwards, forwards or meet-in-the-middle strategies. (U3) Students will understand that problems can be solved using a variety of strategies and knowing the strategies and when they work best make the students better problem solvers.

Acquisition	
Students will Know…	**Students will be able to (DO)…**
(K1) Students will know the following procedures for factoring quadratic polynomials in one variable: • Special factorizations • Guess-and-check (split the middle method) • Using the sum and product of roots (the box method) • Completing the square and using difference of squares (K2) Students will know the following special factorizations: • Perfect squares • Difference of squares (K3) Students will know how to solve quadratic equations using the following methods: • Completing the square • Using the discriminant formula • Factoring • Graphically by using x -intercepts	(D1) Students will be able to factor second degree polynomials in one variable using the following methods: • Special factorizations • Guess-and-check (split the middle method) • Using the sum and product of roots (the box method) • Completing the square and using difference of squares (D2) Students will be able to compare and contrast the methods for factoring polynomials. (D3) Students will be able to recognize, verify and apply special factorization identities. (D4) Students will be able to solve quadratic equations by • Completing the square • Using the discriminant formula • Factoring • Graphically by using x -intercepts

(K4) Students will know that quadratic equations may have two distinct real solutions, one real solution with multiplicity two, or two complex solutions occurring in conjugate pairs. (K5) Students will know that the value of the discriminant can be used to determine the multiplicity and type of solutions. (K6) Students will know the steps to transform between the standard form and the vertex form of a parabola and how to use the vertex form to determine the global extrema. (K8) Students will know the characteristics of the following function families: constant, linear, absolute value, and quadratic. (K9) Students will know the shape of the graph of the following function families: constant, linear, absolute value and quadratic. (K10) Students will know the effects on the graph and algebraic formula of a function of the following transformations: reflection over the x-axis, horizontal and vertical translations, and dilations.	(D5) Students will be able to recognize and describe in English the transformations applied to the graph or algebraic equation of a parent function. (D6) Students will be able to draw the graph and calculate the algebraic equation of a function defined as a list of transformations applied to a parent function. (D7) Students will be able to derive the quadratic formula using completing the square method. (D8) Students will be able to verify algebraic solutions using a graphing utility. (D9) Students will be able to use the graph of a quadratic function to determine: • Range • Intervals on which the function is increasing or decreasing • Global extremes • Intercepts • Zeros • Value of the function at points in the domain (D10) Students will be able to transform between the quadratic equation form and the vertex form of a parabola.

**Note: After each learning target, please provide a parenthetical label (i.e., U1, U2, K1, K2, D1, D2, etc.)

Stage Two: Assessment Plan

Summative Assessment Overview

The students are asked to use the knowledge accumulated in the quadratics unit in a practical situation. To succeed, students must demonstrate the ability to model a real-life scenario using quadratic functions, to solve one or more quadratic equations, to write their work clearly and concisely, and to construct a convincing argument based on math.

In this task, students are asked to calculate the safe following distance to a leading vehicle considering the decelerations of the two vehicles and the driver reaction time. Students are asked to perform the same calculations in two more scenarios, distracted driver, and inclement weather. Students must analyze the results and present a convincing argument for their classmates to slow down and pay attention to the road.

The task involves solving a quadratic and graphing quadratic functions. The students that use an algebraic method will have to use the quadratic formula to solve the equation. The students that use a graphical method to solve the problem will use transformations. To complete the task successfully the students must achieve the following learning targets: U1-U3, K3, K4, K5, K8-K10, D4, D5, D6, D8, D9.

Directions for students

Your task is to convince your friends of the importance of maintaining a safe following distance to a front vehicle. Consider the following scenarios.

Scenario:

1. Your vehicle and a leading vehicle in front of you travel at constant speed of 65 Mph.
2. The driver of the leading vehicle slams the brakes to avoid collision with a deer that just jumped in the road.
3. The leading vehicle decelerates with constant deceleration of $-10\ m/s^2$ until comes to a complete stop.
4. The driver of the follower vehicle has reaction time of $\frac{3}{4}$ seconds before it slams the brakes too and decelerates with a constant deceleration of $-9\ m/s^2$.

Distracted driver scenario:

1. The reaction time is 46% longer.

Wet pavement scenario:

1. The deceleration rate is 60% slower for both vehicles.

Your task is:

1. Brainstorm and make a list of all the variables that can be considered.
2. Create a drawing to summarize the problem. Mark the distances and time on your drawings.
3. Use functions to model the movement of the two vehicles. You may have to use piecewise functions.
4. Use mathematical equations to model the state of the two cars at the moment the follower vehicle comes to a rest.
5. Calculate a minimum safe following distance the follower vehicle should maintain from the leading vehicle such that it reaches a complete stop at a distance of $1m$ from the leading vehicle.
6. Explain your math using diagrams.
7. Use math to explain the importance of traveling at lower speeds in inclement weather.

8. Use math to explain the importance of avoiding distractions while driving.
9. Create your final product. Choose one of
    a. An email for your friend.
    b. A Power Point slideshow.
    c. A video presentation.
    d. A poster.

You will work in teams of four on the assignment for two class blocks. Each member of the team must have at least one contribution for each step of the assignment. You may move on to a next step only after Mrs. Evans checked your work and provided feedback.

Block 1. Step 1. Understand the problem and create a diagram for it.
Block 1. Step 2. Define functions to model the problem.
Block 1. Step 3. Define the mathematical equations.
Block 1. Step 4. Distribute the mathematical work between the team members.

Block 2. Step 5. Solve the equations using any of the methods learned.
Block 2. Step 6. Choose the final product type and divide the work among team members.
Block 2. Step 7. Create the final product.
Block 2. Step 8. Write an individual reflection explaining your contributions to the success of the project.

The project is graded holistically using the following rubric:

4 points – The student made significant contributions to the project, solved the assigned equation correctly, explained and presented the solution clearly and concisely.

3 points – The student made reasonable contributions to the project, solved the assigned equation correctly, but was unable to explain the process clearly.

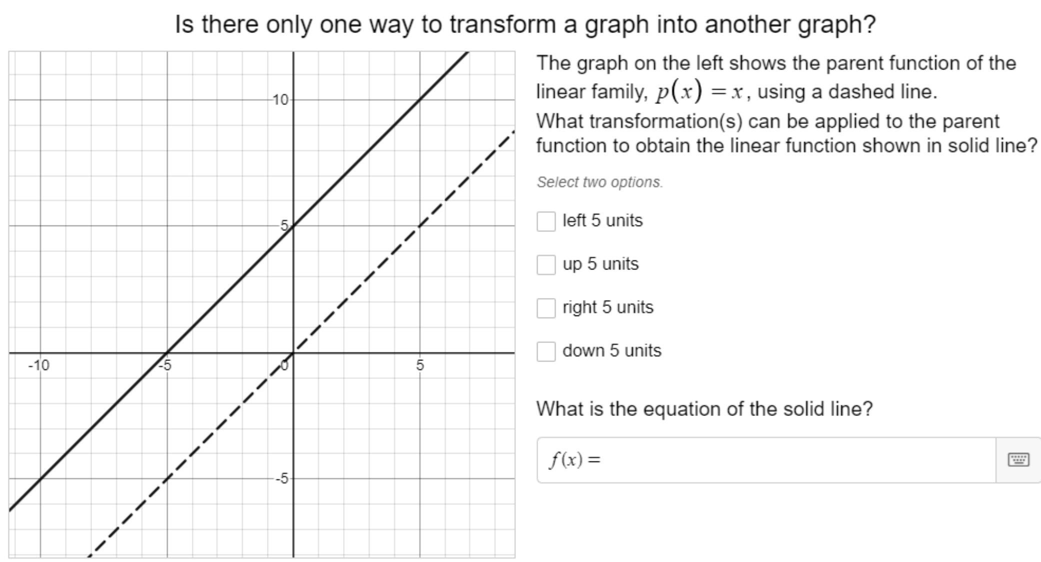
2 points – The student attempted to contribute but made errors in the mathematical process.

1 point – The student had a minimal contribution to the project and did not solve the assigned equation.

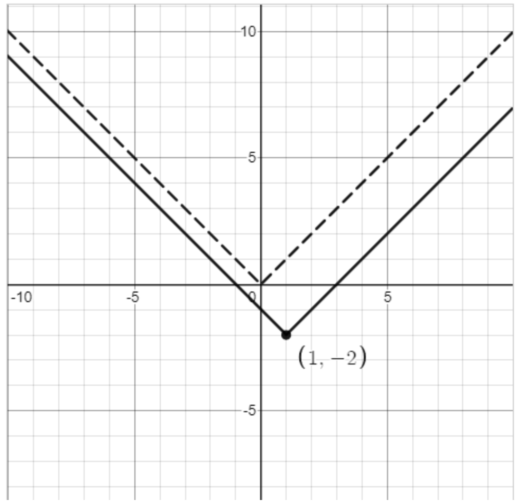
0 points – The student was not able to identify any contribution.

Stage Three: Learning Plan

Lesson/Day	*Learning Target(s) Addressed*	*Summary of Content AND Instructional Plan/Activities*
1	U1	See attached lesson plan. LP_Evans_LessonPlan.docx LP_Evans_Notes.docx LP_Evans_Slides.pptx
2	U2, K6, K8, K9, D5, D6	**Lesson specific learning targets:** (U1) Students will understand that complex systems can be built from standard parts using a simple set of transformations. (K2.1) Students will know that members of the same function family share common characteristics. (K2.2) Students will know that any member of a family can be obtained from a parent function. (D2.1) Students will be able to describe in natural language the transformations applied to obtain a given graph from the graph of the parent graph. The transformations are horizontal and vertical translations, reflection over the x-axis and vertical dilation. (D2.2) Students will be able to sketch the graph of a function given a list of transformations. **Lesson Overview.** In this lesson students learn the definition of function family, apply it to families already studied and the quadratics. The second part of the lesson presents transformations of graphs. **Lesson Steps:** Step 1. Warm-Up (non-graded summative assessment) Step 2. Discuss and complete the Desmos activity assigned in Lesson 1. Step 3. Growth mindset discussion. Step 4. (Lesson/I do). Define function families. Use human family as analogy and ask the students to write down for themselves the definition of family. Step 5. (We do it together). Desmos activity https://teacher.desmos.com/activitybuilder/custom/618017ec1819a028d8fda87d Step 6. (Lesson/I do). Transformations: reflection over the x-axis, horizontal and vertical translation, vertical dilation. Each transformation is motivated using real-life situations. Step 7. (We do it together). Desmos Activity. https://teacher.desmos.com/activitybuilder/custom/6180191e5909391c87c435ee Step 8. (You do it together). Desmos Activity. https://teacher.desmos.com/activitybuilder/custom/61af81bbe509400a3900792f **Resources:** LP_Evans_2_Slides.pptx, LP_Evans_2_Notes.docx

3	U2, K10, D5, D6	**Lesson specific learning targets:** (U3.1) Students will understand that complex equations can be built from parent functions using a set of transformations. (K3.1) Students will know that the equation of any member of a family can be obtained from the equation of the parent function. (D3.1) Students will be able to describe in natural language the transformations applied to a parent function to obtain a given equation. (D3.2) Students will be able to determine the equation of a function given a list of transformations. **Lesson Overview.** This is the second lesson on function families and transformations. While the first lesson showed the effect of the transformations on the graph of the function, this lesson is focused on the changes of the function's equation. The students use what they learned in the previous class to check their solutions. **Lessons Steps.** Step 1. Summative assessment of previous class. Desmos activity. https://teacher.desmos.com/activitybuilder/custom/6180152cefa80a0a1aafc84c Step 2. (Lesson/I do). Use the graphical representation to deduce the new coordinates of a point after each transformation studied. Apply the formula to deduce the equation of the transformed function. Step 3. (Launch/Explore/Discuss). Desmos activity. https://teacher.desmos.com/activitybuilder/custom/61801682d9c8050a01108e8f (Launch) Students are told the goal of the activity. (Explore) The students will work on the activity. Several screens have multiple solutions. (Discuss) The teacher makes the math visible. *(Screen 1) How would you describe the transformation from the dashed to the solid graph?* *There are two possible descriptions: move up by 5 or left by 5.* *The new equation for left translation 5 units is x+5*  *(Screen 2) What if we apply two transformations in a row?* *If we look at the vertex, we see that it moves down 2 and right 1. Does it matter if it moves first down then right, or first right, then down?* *No, it does not matter. If I want to move from one corner of the classroom to the diagonally opposite one it does not matter which walls I walk along, I get to the same point.*

Does the order of transformations matter?



The graph on the left shows the parent function of the linear family, $p(x) = |x|$, using a dashed line.
What two transformations can be applied to the parent function to obtain the linear function shown in solid line?

Select two options.

- ☐ down 2 units, right 1
- ☐ right 1 unit, up 2
- ☐ up 2 units, right 1
- ☐ right 1 unit, down 2

What is the equation of the solid line?

$f(x) =$

(Screen 3) Let’s check that the order does not matter.

$p(x) = |x|$

The equation of p moved right one unit is

$f(x) = |x - 1|$

The equation of f moved down two units is

$g(x) = |x - 1| - 2$

$p(x) = |x|$

The equation of p moved down two units is

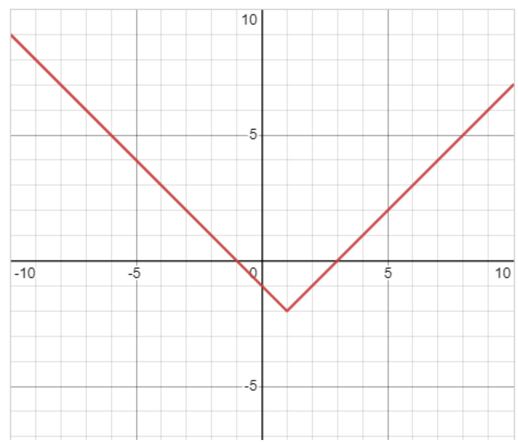
$h(x) = |x| - 2$

The equation of h moved right one unit is

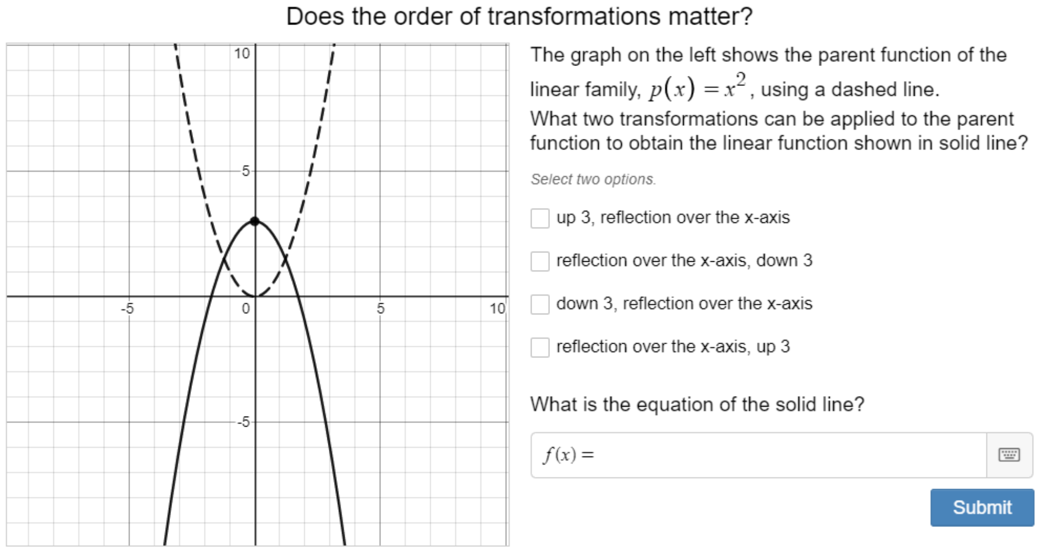
$k(x) =$

Do the functions g and k have the same equation? Why or why not?

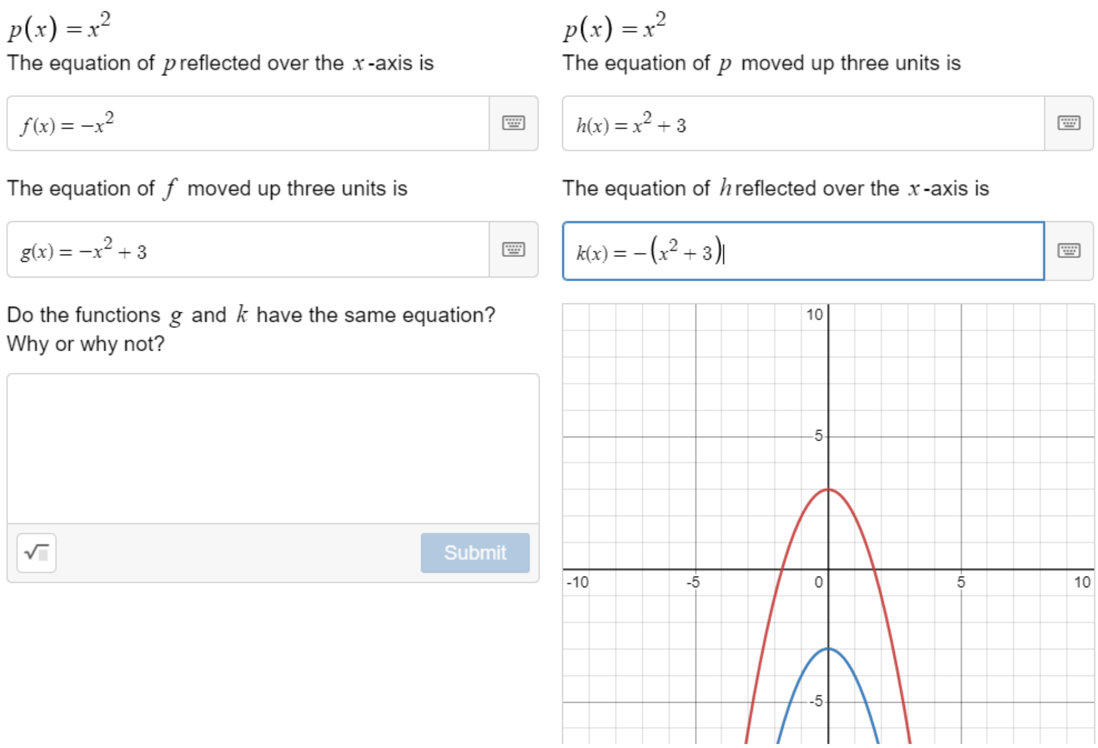
Submit



(Screen 4) Let’s look at different function.



If we do the reflection first, then the second transformation if move up by 3.
If we first do the vertical translation, we have to move down by 3, and then reflect.
(Screen 5) Let's investigate what happens we change the order of reflection and vertical translation.



Step 4: (You do it). Desmos activity or worksheet.
https://teacher.desmos.com/activitybuilder/custom/6181bd14b6624e584613511b

Resources: LP_Evans_3_Notes.docx, LP_Evans_3_Practice.docx, LP_Evans_3_Slides.pptx

4	U3, K1, D1, D2	**Lesson specific learning targets:** (U4.1) Students will understand that sometimes using guess-and-check methods is faster and less error-prone than using an algorithm to solve the problem. (K4.1) Students will know that the factor form of an equation is $f(x) = a(x - p)(x - q)$. (K4.2) Students will know that the zeros of a quadratic in factor form $f(x) = a(x - p)(x - q)$ are $x = p$ and $x = q$. (D4.1) Students will be able to calculate the factor form of a quadratic given its graph. (D4.2) Students will be able to calculate the factor form of a quadratic given its standard form using the split-the-middle method. (D4.3) Students will be able to calculate the standard form of a quadratic given its factor form. **Lesson overview:** This is the fourth lesson in the Quadratic Functions unit. The students learned the standard and vertex form of quadratic functions, properties of quadratic functions and used transformations to obtain a quadratic function from the square function. In this lesson, I introduce the factor form of a quadratic function and teach the split the middle method to factor a quadratic function with the leading coefficient equal to one. Factoring is an important skill that will be used later in context of polynomial and rational functions. The split the middle method is explicitly stated in the curriculum framework for Algebra II. **Lesson steps:** Step 1. Warm-Up/Summative assessment (Launch) I will ask the students to complete page 1 of the notes handout. (Explore) I will give the students 15 minutes to work on their own and then I will review the questions with the entire class. As the students work on the warm-up, I will walk around the classroom to check on their progress and answer questions. I will record frequently made mistakes. If most students make good progress, then I will ask specific students to write answers on the white board. (Discuss) If the students struggle with the assessment, we will solve together the exercises. If students already wrote the answers, then I will show only the answers to the last problem. Step 2. (I do/We do.) Factor form lesson. First I model solving an exercise, and then we do another one together. Step 3. (You do) Desmos activity or worksheet. https://teacher.desmos.com/activitybuilder/custom/618841e246297e09dbad4f2b **Resources:** LP_Evans_4_Notes.docx, LP_Evans_4_Practice.docx, LP_Evans_4_Slides.pptx
5	U3, K1, K2, D1, D2, D3	**Lesson overview:** In lesson 5, students work in groups to deduce and apply special factorization formulas. **Lesson steps:** Step 1. Warm-up/Summative assessment Step 2. (Launch/Explore/Discuss) see instructions in the notes document. Step 3. Quiz from lessons 1,2,3. **Resources:** LP_Evans_5_Notes.docx, LP_Evans_5_Practice.docx

6	U3, K3, K4, K5, D7, D8, D4,	**Lesson specific learning targets:** (K6.1) Students will know that quadratic equations may have two distinct real solutions, one real solution with multiplicity two, or two complex solutions. (K6.1) Students will know that a quadratic function whose graph does not intersect the x-axis has roots with imaginary components. (K6.3) Students will know that the value of the discriminant can be used to determine the multiplicity and type of solutions. (K6.4) Students will know that complex solutions occur in conjugate pairs. (D6.1) Students will be able to solve quadratic equations using the quadratic formula. **Lesson overview.** This is the last lesson before the review and test of the Quadratic Functions unit. In this lesson, the students learn the quadratic formula and its applications in solving quadratic equations. **Lesson Steps.** Step 1. Warm-up/Summative assessment Step 2. Lesson (I do. We do it together.) Step 3. You do it. **Resources:** LP_Evans_6_Notes.docx, LP_Evans_6_Practice.docx, LP_Evans_6_Slides.pptx
7,8		Quiz lessons 4,5,6. Work on the performance task (see Summative Assessment section).
9		Unit review. See LP_Evans_SummativeAssessmentReview.docx.
10		Summative assessment (test). See LP_Evans_SummativeAssessment.docx and LP_Evans_SummativeAssessmentGradingRubric.docx