Review of the Quadratic Functions Unit

Quadratics Equations - Summative Assessment		
Topic	**Number of lessons**	**Questions**
Definitions and Graphing	1	2/16
Families of Functions	2	4/16
Factoring	3	6/16
Quadratic Equations	2	4/16

In each of the tables that follow, you will find first a set of notes to help you remember the class content, and one pairs of one solved exercise and one practice exercise.

Properties of Parabolas

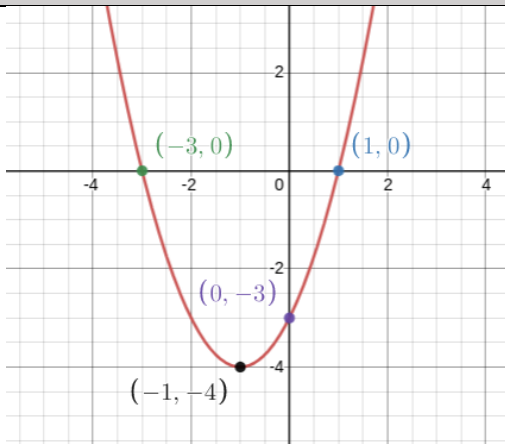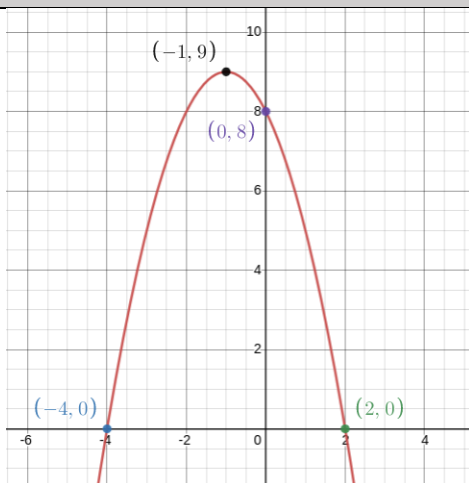| Review |
|---|
| The **standard form** of a quadratic equation is $f(x) = ax^2 + bx + c$, $a \neq 0$. |
| **x-intercepts** are the points where the graph intersects the x-axis. |
| **y-intercept** is the point where the graph intersects the y-axis. |
| The **vertex** of a parabola in standard form has the coordinates $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. |
| The **axis of symmetry** is $x = -\frac{b}{2a}$. |

Solved Question	
	Question 1: Fill in the blanks. 1. The parabola opens ___up___. 2. The x-intercepts are ___(-3,0), (1,0)___. 3. The y-intercept is ___(0,-3)___ 4. The vertex is ___(-1,-4)___ 5. The axis of symmetry is ___$x = -1$___

Practice Question	
	Fill in the blanks. 1. The parabola opens _____ 2. The x-intercepts are _____ 3. The y-intercept is _____ 4. The vertex is _____ 5. The axis of symmetry is _____

Vertex Form

Review

The **vertex form** of a quadratic has an equation of the form $f(x) = (x - h)^2 + k$, where (h, k) is the vertex of the parabola.
 	
Procedure to convert from standard form to vertex form:
 	
   1. Identify the constants a (the number before x^2), b (the number before x), c (the number with no x next to it).
   2. Calculate $-\frac{b}{2a}$.
   3. Calculate $f\left(-\frac{b}{2a}\right)$ by replacing x with the number calculated in Step 2.
   4. Write the vertex form equation by replacing h with the number in Step 2 and k with the number in Step 3.

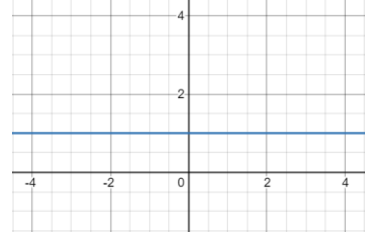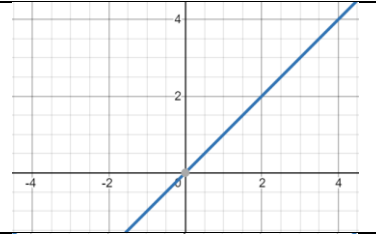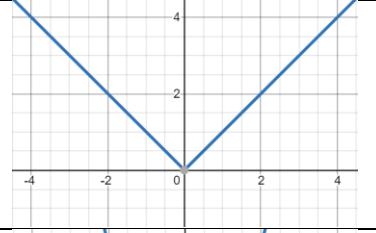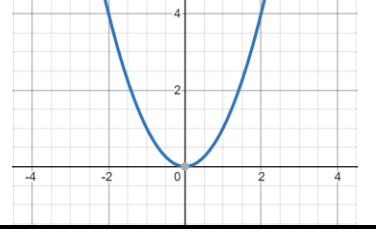| Solved Question |
|---|
| **Question 2:** Transform the equation in the standard form $f(x) = x^2 - 2x + 5$ to the vertex form.

Solution:
Step 1. $a = 1, b = -2, c = 5$.
Step 2. $-\frac{b}{2a} = -\frac{-2}{2} = 1$
Step 3. $f(1) = 1^2 - 2 \cdot 1 + 5 = 4$
Step 4. The vertex form of the function is $f(x) = (x - 1)^2 + 4$. |

Practice Question
 Question 2: Transform the equation in the standard form $f(x) = -x^2 - 2x + 1$ to the vertex form.

Function families and Transformations

		Review	
Family Name	**Parent Function**	**Graph**	**Characteristics**
Constant	$f(x) = 1$		The graph is a horizontal line. Domain: all real numbers Range: one value
Linear	$f(x) = x$		The graph is a slanted line (not horizontal, not vertical) Domain: all real numbers Range: all real numbers.
Absolute Value	$f(x) = |x|$		The graph is V-shaped. Domain: all real numbers. Range: $[0, \infty)$
Quadratic	$f(x) = x^2$		The graph is U-shaped, it is called a parabola. Domain: all real numbers Range: $[0, \infty)$

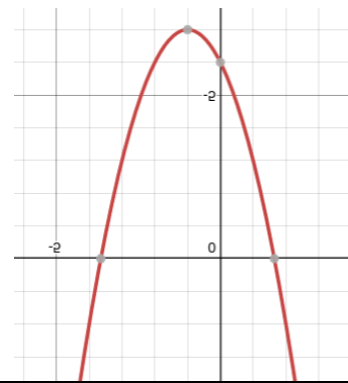| Transformation		Graph	Equation
Reflection			$f(x) \to -f(x)$
Horizontal Translation	Move Right		$x \to x - b$
	Move Left		$x \to x + b$
Vertical Translation	Move Up		$f(x) \to f(x) + a$
	Move Down		$f(x) \to f(x) - a$
Vertical Dilation	Vertical Stretch		$f(x) \to df(x), d > 1$
	Vertical Compression		$f(x) \to cf(x), c < 1$

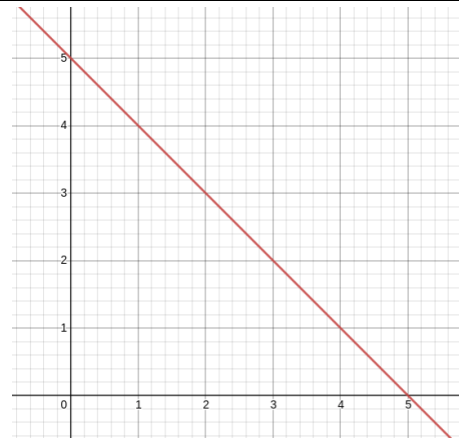| Solved Questions |
|---|
| **Question 3. A)** To what family does the graph to the right belong?
Solution: *the quadratic family.*

B) What is the domain and range of the function?
Solution. *The domain is all real numbers, and the range is* $(-\infty, 7]$ |
| **Question 4**. Consider the graph to the right.

A. What function family does it belongs to?
Solution: *The linear family.*
B. Write the equation of the parent function of the family.
Solution: $p(x) = x$
C. List, in order, the transformations that can be applied to the parent function to obtain the given graph.
Solution:
Reflection over the x-axis, move right 5 units. |
| **Question 5.** Consider the equation $f(x) = -\lvert x - 1\rvert$

A) What function family does it belongs to?

Answer: *The absolute value family*

B) Write the equation of the parent function of the family.

Answer: $p(x) = \lvert x \rvert$

C) List, in order, the transformations that can be applied to the parent function to obtain the given equation.

Answer: *Reflection over the x-axis, right one unit.* |
| **Question 6.** Write the equation of the function obtained from $p(x) = x$ by performing the following transformations

    1. Reflection over the x-axis
    2. Vertical dilation by a factor of 3 |

3. Left translation by 5 units
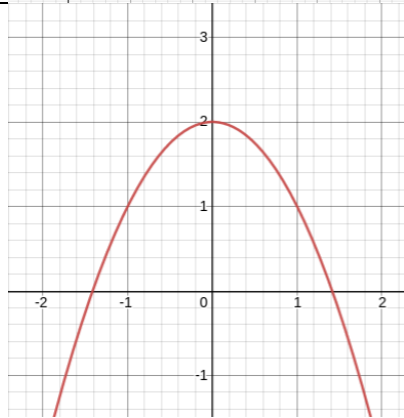
Solution:

1. $f(x) = -x$
2. $g(x) = -3x$
3. $h(x) = -3(x+5) = -3x - 15$

Practice Question

Question 3. To what family does the graph below belong? What is the domain and range of the function?
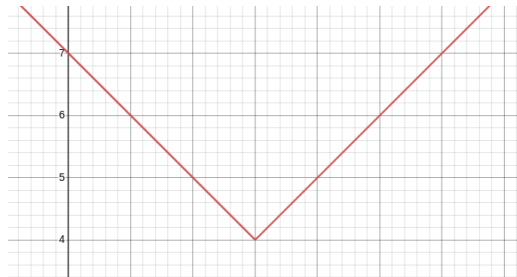
Answer:

Question 4. Consider the graph to the left.

A) What function family does it belongs to?

Answer:

B) Write the equation of the parent function of the family.

Answer:

C) List, in order, the transformations that can be applied to the parent function to obtain the given graph.

Answer:

Question 5. Consider the equation $f(x) = (x - 1)^2 + 3$

   A) What function family does it belongs to?

   Answer:

   B) Write the equation of the parent function of the family.

   Answer:

   C) List, in order, the transformations that can be applied to the parent function to obtain the given equation.
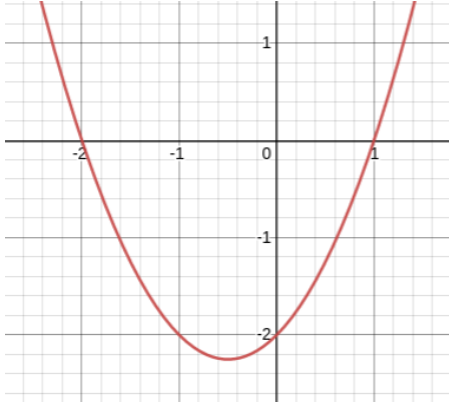
   Answer:

Question 6. Write the equation of the function obtained from $p(x) = x$ by performing the following transformations

   1. Move up 3 units
   2. Reflection over the x-axis
   3. Left translation by 5 units

Answer:

Factor Form

Review

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Graph Method

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Step 1. Take the x-coordinates of the x-intercepts.
Step 2. The factors of the factor form are x minus coordinate.

Example:
$$f(x) = \big(x - (-2)\big)(x - 1)$$
$$= (x + 2)(x + 1)$$

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Guess and check

Step 1. Given the equation, guess a factor.

Step 2. Split the middle term so that there you can factor out guessed binomial.

Example:

$f(x) = x^2 + 4x + 3$

Step 1. Guess that $x + 1$ is a factor

Step 2. $f(x) = x^2 + x + 3x + 3 = x(x + 1) + 3(x + 1) = (x + 3)(x + 1)$

Split the middle

Step 1. List pairs of numbers that multiply to the constant term (c).

Step 2. Check which pair add up the negative of the coefficient of the linear term $(-b)$.

Step 3. Write the factor form.

Example:

$f(x) = x^2 + 4x + 3$

Step 1. Pairs that multiply to 3 are $1 \; and \; 3, -1 \; and -3$

Step 2. The pair $-1 \; and -3$ adds up to -4.

Step 3. The factor form is $f(x) = \big(x - (-1)\big)\big(x - (-3)\big) = (x + 1)(x + 3)$

Special Factorizations

Recognize the right-hand side of one of the special factorizations.
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$
$(a + b)(a - b) = a^2 - b^2$

Example:

Factor the function $f(x) = x^2 - 4$.

Use the difference of squares formula to get $f(x) = x^2 - 2^2 = (x + 2)(x - 2)$

Quadratic Formula

Use the quadratic formula to calculate the roots of the function $f(x) = ax^2 + bx + c$. Then the factors are binomials x minus root.

1. Identify the constants a (the number before x^2) , b (the number before x) , c (the number with no x next to it).
2. Calculate the roots using the quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Factor the function $f(x) = 2x^2 + 3x - 5$

Step 1. $a = 2, b = 3, c = -5$

Step 2. The solutions are

$$\frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot (-5)}}{2 \cdot 2} = \frac{-3 \pm \sqrt{9 + 40}}{4} = \frac{-3 \pm 7}{4}$$

The solutions are:

$$\frac{-3 - 7}{4} = -\frac{10}{4} = -\frac{5}{2}$$

$$\frac{-3 + 7}{4} = -\frac{4}{4} = -1$$

Practice Questions

Question 7. Factor the function $f(x) = x^2 + 8x + 15$.

Question 8. Factor the function $f(x) = x^2 - 100$.

Question 9. Factor the function $f(x) = -49x^2 + 100$.

Question 10. Factor the function $f(x) = x^2 + 5x$.

Question 11. Factor the function $f(x) = 10x^2 - 11x + 1$

Question 12. Factor the function $f(x) = x^2 + x - 6$

Quadratic Equations

Practice Problems
The same methods used to factor a quadratic can be used to solve a quadratic equation.
Question 13. $(x - 200)(2x - 1000) = 0$
Question 14. $x^2 + 2x + 1 = 0$

Question 15. $x^2 - x + 5 = 0$

Question 16. $x^2 + 4x + 3 = 0$