Unit 3. Section 4. Factor Form.

Warm-up.

Question 1. A *factor* of a number is a number that divides that number without a remainder. Write the factors of the following numbers:

(A) 24

(B) 15

(C) 7

Question 2. Yes No. Is 27 a factor of 81? Why or why not?

Question 3. What pairs of integers multiply to 12?

Question 4. What pairs of integers multiply to -6?

Question 5. Complete the following factorizations.

(A) $x^2 + 2x = x(\_\_\_\_\_\_\_\_)$

(B) $5x^2 + 10x = 5x(\_\_\_\_\_\_\_\_)$

(C) $x(x+1) + 2(x+1) = (\_\_\_\_\_\_\_\_\_\_)(x+1)$

(D) $3x(2x+1) + 5(2x+1) = (\_\_\_\_\_\_\_\_\_\_)(2x+1)$

Forms of quadratic functions

Form Name	Form Equation	Examples
Standard	$f(x) = ax^2 + bx + c, a \neq 0$	$f(x) = x^2 + x - 3$
Vertex	$f(x) = a(x - h)^2 + k, a \neq 0$	$f(x) = (x - 1)^2 - 3$
Factored	$f(x) = a(x - p)(x - q), a \neq 0$	$f(x) = (x - 2)(x + 3)$

Practice 1. Given the function $f(x) = 3(x - 1)(x - 2)$ in factor form, what are the values for the constants a, p, q?

$a =$__ $p =$__ $q =$__

What is the standard form of the function f?

Method 1: *The crab claws*

$(x - 1)(x - 2)$

= ________________

= ________________

$f(x) =$ ______________

Method 2: *The box method*

	x	-2
x		
-1		

Conversions between the standard form and factor form

To convert the equation of a quadratic function from the factor form, we use the distributive property and simplification using the following steps:

1. Multiply the binomials $x - p$ and $x - q$.
2. Simplify the expression by adding the x-terms.
3. Multiply the trinomial from step 2 by a.

To convert the equation of a quadratic function in standard form to factor form is called **factoring**. We will learn several strategies to factor.

Split the Middle

We are given a function in standard form. For example, $f(x) = x^2 - 8x + 12$.

Our goal is to calculate p and q such that $f(x) = x^2 - 8x + 12 = (x - p)(x - q)$

First, we simplify the right-hand side of the equation.

Method 1: *The crab claws*

$(x - p)(x - q)$

$= x^2 - (p + q)x + pq$

Method 2: *The square method*

	x	$-q$
x	x^2	$-qx$
$-p$	$-px$	pq

Set the two equations equal.

$x^2 - (p + q)x + pq =$

These are equal if the coefficients of x^2, x and the constant term are equal.

$-(p + q) =$

3.4. Factor Form

$pq =$

Search for pairs of numbers that multiply to 12 and add up to 8.

Pair	Sum
1, ____	13
2,____	
3, ____	
4, _____	

We found $p =$ ___ and $q =$ ____

The factor form is $f(x) =$ __________

Check the result

General strategy

Given a quadratic function in standard form with $a = 1$, $f(x) = x^2 + bx + c$, to convert the function to the factored form we

1. List pairs of numbers that multiply to c.
2. If a pair adds up to a number different than $-b$ we discard the pair.
3. If a pair adds up to $-b$ we keep the pair. We will use the variables p and q to denote the pair.
4. The factor form is $f(x) = (x - p)(x - q)$.

Suggestions to reduce the number of pairs checked:

	$pq > 0$	$pq < 0$
$p + q > 0$	Check positive divisors of pq.	Check one positive and one negative divisor with the positive one greater than the absolute value of the negative divisor.
$p + q < 0$	Check negative divisors of pq.	Check one positive and one negative divisor with the positive one smaller than the absolute value of the negative divisor.